Appendix D

A Guide to the Literature

Here we list and briefly describe a number of textbooks, survey articles, and more specialized books which the reader may find useful. We haven't aimed for comprehensive coverage. Rather, we've commented on the sources the reader is most likely to run into, provided pointers to topics not discussed in this book (in particular, modal proof theory and theorem proving, and first-order modal logic) and drawn attention to some interesting emerging themes.

This is a good place to mention the *Advances in Modal Logic* initiative, which attempts to bring together scholars working in various areas of modal logic and its applications. You can find out more at:

`http://www.aiml.net`

The collection *Advances in Modal Logic, Volume 1*, edited by Kracht, de Rijke, Wansing and Zakharyaschev [288], contains a selection of papers from the first conference hosted by the initiative. A selection of papers from the second workshop, *Advances in Modal Logic, Volume 2*, edited by Zakharyaschev, Segerberg, de Rijke, and Wansing [461], has also appeared, while a selection from the third workshop was in preparation at the time of writing.

Textbooks on Modal Logic

To start, here's an annotated list of textbooks on modal logic.

- *A Manual of Intensional Logic*, van Benthem [43]. What is modal logic? What isn't! This inspiring little book takes the reader on a whirlwind tour of the many faces of modal logic. The book is deceptively easy to read; alert readers will soon cotton onto the fact that the author indicates unexplored territory on practically every page.
- *The Logic of Provability*, Boolos [67]. Clear, up-to-date, introduction to provability logic.

- *Modal Logic*, Chagrov and Zakharyaschev [86]. A recent advanced textbook on modal logics in the basic modal language and their connections with super-intuitionistic logics. Concentrates on the fine structure of the lattice of normal modal logics, using methods not covered by our book, to prove general results on various properties of logics.
- *Modal Logic. An Introduction*, Chellas [90]. A readable introductory text which focuses on completeness-via-canonicity and decidability-via-filtration for the basic modal language. Also introduces neighborhood (or Montague-Scott) semantics, a tool for analyzing non-normal modal logics (Chellas calls neighborhood models minimal models).
- *Proof Methods for Modal and Intuitionistic Logic*, Fitting [145]. This beautiful book explores in detail a number of proof methods (tableaux, sequent systems, natural deduction) for modal logic; we have no hesitation in recommending this classic work as a great source for finding out more about modal proof theory.
- *Types, Tableaus, and Goedel's God*, Fitting [147]. An introduction to higher-order modal logic that ends by formalizing Goedel's ontological argument for the existence of God! Quite apart from anything else, it is one of the best introductions to ordinary (non-modal) higher-order logic around. Tableaux based.
- *First-Order Modal Logic*, Fitting and Mendelsohn [148]. An excellent introduction to first-order modal logic. Addresses both technical and philosophical aspects of quantification and equality. Provides both Hilbert-style and tableaux proof systems.
- *Logics of Time and Computation*, Goldblatt [183]. Clearly written intermediate level text which focuses on completeness results for the basic modal and temporal languages, and a variety of extended modal languages, including Until-based temporal languages, PDL, and first-order PDL. A useful book to have around.
- *Mathematics of Modality*, Goldblatt [184]. This book brings together a number of the author's papers on modal logics. In particular, it contains his seminal PhD thesis which can be hard to obtain in its article version [190, 191].
- *Dynamic Logic*, Harel, Kozen and Tiuryn [218]. A detailed, well written, introduction to propositional dynamic logic, and many of its extensions, including first-order dynamic logic. A good choice for readers wanting to find out more about this important branch of modal logic.
- *A Companion to Modal Logic*, Hughes and Cresswell [240]. This was the first textbook to move beyond the staples of the classical period (relational semantics, canonical models, filtrations) and discuss distinctively modern topics (notably frame incompleteness). Short, accessible, and clear, it is still a valuable introductory text, though it has has since been superseded by the next entry.
- *A New Introduction to Modal Logic*, Hughes and Cresswell [241]. An admirably clear and wide ranging introductory text. Although many topics (such as the

standard translation) are not discussed, it manages to at least mention many modern themes such as frame definability and extended modal languages. Contains a good discussion of first-order modal logic.

- *Tools and Techniques in Modal Logic*, Kracht [284]. Advanced book on mathematical aspects of modal logic, taking a polymodal perspective. Contains an in-depth study of correspondence and completeness, duality theory, the lattice of modal logics, and transfer results from monomodal to polymodal logics.
- *The "Lemmon Notes": An Introduction to Modal Logic*, Lemon and Scott [303]. All that exists of an unfinished monograph on modal logic (Lemmon's death in 1966 prevented its completion). The original source for work on filtrations and canonical models, for many years it was *the* definitive introduction to modal logic. Although out of date, its quality still shines through.
- *Epistemic Logic for AI and Computer Science*, Meyer and Van der Hoek [334]. An introductory text on epistemic logic. Covers the basic modal approach, as well as more advanced models and default reasoning.
- *A Short Introduction to Modal Logic*, Mints [336]. This little book is a short introduction to Gentzen-style proof systems for **S5**, **S4** and **T**. A useful starting point in modal proof theory.
- *First Steps in Modal Logic*, Popkorn [365]. If you find the present book too difficult and feel the need to consult something simpler, we suggest you try Popkorn's text. Like the present book it is semantically oriented and takes for granted that modal logic has more to offer than the basic modal language. It is clearly written, mathematically precise, and besides the present book, it is the only textbook we know that discusses bisimulations.
- *Self-Reference and Modal Logic*, Smoryński [409]. The classic introduction to provability logic. Beautifully written. More demanding than the Boolos volume.
- *From Modal Logic to Deductive Databases*, edited by Thayse [422]. Wide-ranging introduction to modal logic. Discusses links with natural language, temporal reasoning, various forms of defeasable reasoning, and deductive databases.

Books in Other Languages

Next, here's a list of books in languages other than English, without comments.

- *Essai de Logique Déontique*, Bailhache [17].
- *Logicaboek*, Batens [29].
- *Fondements Logiques du Raisonnement Contextuel. Une Etude sur les Logiques des Conditionnels*, Crocco [98].
- *Logica, Significato e Intelligenza Artificiale*, Frixione [151].
- *La Logique du Temps*, Gardies [168].
- *Essai sur les Logiques des Modalités*, Gardies [169].

- *Logique. Volume 3. Méthodes pour l'intelligence artificielle*, Gochet, Gribomont, and Thayse [180].
- *Una Introducción a la Lógica Modal*, Jansana [250].
- *La Logique Déductive*, Kalinowski [262].
- *Glauben, Wissen und Wahrscheinlichkeit. Systeme der Epistemischen Logik*, Lenzen [305].
- *Pour une Logique du Sens*, Martin [321].
- *Forcing et Sémantique de Kripke-Joyal*, Moens [337].
- *Essais sur les Logiques non Chrysipiennes*, Moisil [338].
- *Klassische und nichtklassische Aussagenlogik*, Rautenberg [375].

Survey Articles

Note that virtually all the survey articles listed below are drawn from the following sources:

- *Handbook of Automated Reasoning*, edited by Robinson and Voronkov [380].
- *Handbook of Logic and Language*, edited by Van Benthem and Ter Meulen [49].
- *Handbook of Logic in Artificial Intelligence and Logic Programming*, edited by Gabbay, Hogger, and Robinson [165, 166].
- *Handbook of Logic in Computer Science*, edited by Abramsky, Gabbay, and Maibaum [1].
- *Handbook of Philosophical Logic*, edited by Gabbay and Guenthner [162].
- *Handbook of Proof Theory*, edited by Buss [81].
- *Handbook of Tableau Methods*, edited by D'Agostino, Gabbay, Hähnle, and Posegga [100].
- *Handbook of Theoretical Computer Science*, edited by Van Leeuwen [300].

In fact, these handbooks contain surveys of many other topics in, or related to, modal logic including: auto-epistemic logic, belief revision, combinations of tense and modality, computational treatments of time, conditional logic, decision procedures, deontic logic, description logics, epistemic aspects of databases, general decidable fragments, higher-order modal logic, logics of programs, non-monotonic temporal reasoning, philosophical perspectives on first-order modal logic, provability logic, reasoning about knowledge, time and change in AI.

- *Correspondence Theory*, van Benthem [42]. For many years this was the only easily accessible general reference on correspondence theory, and it is still well worth reading. Lots of telling examples, and propelled by a clear vision of what the modal enterprise is all about.
- *Temporal Logic*, van Benthem [46]. A wide-ranging and thoughtful discussion of key themes in temporal logic.

- *Basic Modal Logic*, Bull and Segerberg [73]. An interesting survey, rich in historical detail, which provides a useful point of entry to a topic barely touched on in the present book: the fine structure of the lattice of normal modal logics in the basic modal language.
- *Basic Tense Logic*, Burgess [77]. An accessible survey, mostly devoted to the basic temporal language, but touching on Until-based logics and multi-dimensional systems. Contains many useful examples of step-by-step completeness proofs.
- *Advanced Modal Logic*, Chagrov, Wolter and Zakharyaschev [85]. Takes up the story where the Bull and Segerberg survey leaves off. Strong on the fine structure of the lattice of normal modal logics.
- *Reasoning in Description Logics*, Donini, Lenzerini, Nardi, and Schaerf [123]. Approachable overview article which discusses four types of reasoning important in description logic. Both inference techniques and complexity results are covered.
- *Temporal and Modal Logic*, Emerson [129]. A detailed introduction to temporal logic from the perspective of theoretical computer science. Somewhat dated, but still a good introduction.
- *Basic Modal Logic*, Fitting [146]. An extremely clear introductory survey. Starts with a good discussion of the basic modal language (including Hilbert systems, natural deduction, tableaux methods, the standard translation, and alternative semantics), and then goes on to examine first-order modal logic.
- *Varieties of Complex Algebras* and *Algebraic Polymodal Logic*, Goldblatt [192, 187]. Both of these provide an introduction to the study of varieties of BAOs, emphasizing their connections with modal logics, and focusing on structural properties (such as canonicity) that are related to natural properties of modal logical systems.
- *Tableau Methods for Modal and Temporal Logics*, Goré [199]. Detailed survey of tableaux based proof methods for temporal languages.
- *Dynamic Logic*, Harel [215]. If you want to learn more about PDL, this is a good place to look, though it is more densely written than Harel, Kozen and Tiuryn [218]. Discusses a wide range of variants and extensions of PDL.
- *The Logic of Provability*, Japaridze and de Jongh [251]. A thorough overview of provability logic, covering propositional provability logic, interpretability logic and related areas, as well as predicate provability logic.
- *A Survey of Boolean Algebras with Operators*, Jónsson [258]. Gives an algebraic introduction to the theory of Boolean algebras with operators. The papers by Goldblatt and this one by Jónsson are highly recommended to readers who want to go deeper into the algebraic side of modal logic than our Chapter 5 can offer.
- *Logics of Programs*, Kozen and Tiuryn [280]. Essentially (though not entirely) an introduction to PDL. Useful, but Harel, Kozen and Tiuryn [218] is probably a better choice.

- *Resolution Decision Procedures*, Leitsch, Fermüller and Tammet [301]. An extensive survey of resolution-based decision procedures for fragments of first-order logic. Covers procedures for modal and guarded fragments and fragments corresponding to description logics.
- *Encoding Non-Classical Logics in Classical Logic*, Ohlbach, de Rijke, Nonnengart, and Gabbay [351]. A computationally oriented overview of translation methods for mapping modal and modal-like languages into first-order logic. Discusses various flavors of the standard (relational) translation and the functional translation, from the point of view of both expressive power and decidability.
- *Feature Logics*, Rounds [386]. Authoritative survey of feature logic. Devotes a lot of attention to the modal aspects of feature logic.
- *Modal and Temporal Logics*, Stirling [417]. An extremely useful, technically oriented, article which will appeal to many readers of this volume. Starting with many of the same tools introduced in this book (transition systems, bisimulations, correspondence theory, unraveling), it goes on to to discuss many other topics such as modal μ-calculi and links with automata theory.
- *An Overview of Interpretability Logic*, Visser [444]. An elegant overview of interpretability logic, an extension of provability logic which deals with interpretations between formal theories.
- *On Canonical Formulas*, Zakharyaschev [459]. Canonical formulas are an important frame-theoretic approach to the classification of modal formulas. This is an accessible survey, by the inventor of the method, with plenty of motivations, examples and definitions.

Other Books

- *Vicious Circles*, Barwise and Moss [26]. An introduction to non-well-founded set theory, a set theory where bisimulation rather than extensionality is the key concept. The authors advocate using infinitary modal logic to study such structures, and the area is becoming a research area in its own right.
- *Modal Logic and Classical Logic*, van Benthem [41]. This book (a reworking of the author's PhD thesis) is the primary source on what is now called correspondence theory. Although most of the book is devoted to frame definability, this is also where bisimulations (under the name p-relations) were introduced and shown to capture modal expressivity over models. It may be hard to get hold of, but it is well worth making the effort.
- *Logic of Time*, van Benthem [44]. Explores temporal logic model-theoretically; modal logic is just one strand in the story. The book makes many interesting technical contributions (notably in the study of interval structures) but its real importance is methodological: it's by far the most convincing demonstration

we know of the light model-theoretic methods can throw on difficult conceptual issues.

- *Reasoning about Knowledge*, Fagin, Halpern, Moses and Vardi [133]. Looks set to be the key reference on epistemic logic for quite some time to come. Treats a wide range of topics from a modern semantically-oriented perspective. Clearly written. Highly recommended.
- *Labelled Deductive Systems*, Gabbay [163]. This is not a book on modal logic, but an introduction to a proof theoretical methodology. But modal logics provide many of the nicest examples of the labelling method in action, and if you are interested in modal proof theory you need to know about this.
- *Temporal Logics. Mathematical Foundations and Computational Aspects. Volume 1*, Gabbay, Hodkinson and Reynolds [163]. The first of three projected volumes, this book discusses basic completeness and incompleteness, the temporal logics of special structures (such as $\mathbb{R}$), multi-dimensional temporal logic, fixed-point logic and propositional quantification, and much else besides. Contains a detailed discussion of expressive completeness.
- *Cylindric Algebras, part I & II*, by Henkin, Monk and Tarski. The definitive work on cylindric algebras, it also contains a wealth of results on other branches of algebraic logic and on boolean algebras with operators in general.
- *Relation Algebras by Games*, by Hirsch and Hodkinson. Still in manuscript form at the time of writing, this work promises to become a standard reference on relation algebras. Strong on the connections between relation algebras and model theory and a convincing argument for the application of games in algebraic logic.
- *Temporal Logic of Reactive and Concurrent Systems: Specification*, Manna and Pnueli [318]. Textbook aimed at computer scientists interested in specifying reactive systems. All the necessary temporal logic is introduced and explained in the course of the book.
- *Temporal Verification of Reactive Systems: Safety.*, Manna and Pnueli [319]. Follow up to the previous volume, emphasizing safety issues.
- *Multi-Dimensional Modal Logic*, Marx and Venema [163]. The key reference for work in multi-dimensional modal logic. Discusses two-dimensional modal logics, arrow logics, modal logics of intervals, modal logics of relations, and the problem of defining ‘concrete’ semantics (in the sense of Henkin) for arbitrary modal languages.
- *Past, Present and Future*, Prior [369]. Arthur Prior seems to be one of those authors more often cited than read. This is pity: if you push on past his use of Polish notation you will discover a fascinating writer with a surprisingly up-to-date range of concerns.
- *Set Theory and the Continuum Problem*, Smullyan and Fitting [411]. So why is this one here? Because in Part III of the book the authors show how Cohen-style

forcing arguments can be formulated using first-order **S4**. Roughly speaking, they prove independence results using sophisticated step-by-step arguments.

- *Proof Theory of Modal Logic*, Wansing [447]. A collection of papers devoted to proof-theoretical aspects of modal logic, covering many flavors of proof theory: sequents, resolution, tableaux, display calculi, and translation-based approaches.
- *Displaying Modal Logic*, Wansing [448]. A proof-theoretical monograph that shows in detail how to define display calculi for modal logics.
- *Modal Logic: The Lewis Modal Systems*, Zeman [466]. Provides sequent proof systems for the Lewis systems and some others. Written by a student of Arthur Prior, it draws on some of Prior's unpublished classroom material. Clearly written. Uses Polish notation.
- *Temporal Logic*, Øhrstrom and Hasle [352]. Introduction to temporal logic from a historical perspective. A good way of getting to grips with Prior's work.

The Ω-bibliography

Finally, a fairly comprehensive bibliography of books and articles on modal (and various non-classical) logics is provided by Volume II of the Ω-bibliography on mathematical logic. The references in this volume, edited by Rautenberg [377], stop somewhere in the 1980s; nevertheless, this work can be the starting point of a fascinating quest for treasures in the earlier literature of modal logic.