Logics for Computation

Lecture #5: About Trees, and How to Cut Them

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- We have discussed when two models are the same.
- We have seen an algorithm to check whether a formula is true in a given model.

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- What about the $\langle R \rangle$ language?

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- The solution is: labels!
- They will help us keep track of what is going on in each point in our model.

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for t a new label

- Start with the labelled formula $s{:}\neg\langle R\rangle\varphi$. If there is an R-successor t, then φ should not hold at t.

$$\frac{\begin{array}{c} s{:}\neg\langle R\rangle\varphi \\ sRt \end{array}}{t{:}\neg\varphi} \; (\neg\langle R\rangle)$$

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$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

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- Which are the similarities/differences with tableaux for PL?

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$$\frac{\begin{array}{c} s{:}\neg\langle R\rangle\varphi \\ sRt \end{array}}{t{:}\neg\varphi}$$

$$\begin{array}{c}
s{:}(\neg\langle R\rangle p \wedge (\langle R\rangle q \wedge \langle R\rangle p)) \\
s{:}\neg\langle R\rangle p \\
s{:}(\langle R\rangle q \wedge \langle R\rangle p) \\
s{:}\langle R\rangle q \\
s{:}\langle R\rangle p \\
sRt \\
t{:}q \\
sRu \\
u{:}p \\
t{:}\neg p \\
u{:}\neg p
\end{array}$$

contradiction!!!

- Which are the similarities/differences with tableaux for PL?
- How do we know that we got it right?

The Complete Cast, plus an Example

$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

$$\frac{s{:}\neg(\varphi \wedge \psi)}{s{:}\neg\varphi \qquad\qquad s{:}\neg\psi}$$

$$\frac{s{:}\neg\neg\varphi}{s{:}\varphi}$$

$$\frac{s{:}\langle R\rangle\varphi}{\begin{array}{c} sRt \\ t{:}\varphi \end{array}}$$

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A Closer Look

A Closer Look

- Which similarities / differences with tableaux for PL?

$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

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- Did we get it right?
  - Did we get it right in the PL case, to start with?! Consider the rule:

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  - Did we get it right in the PL case, to start with?! Consider the rule:
- What can we learn from the calculus?
  - Something about models!

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Tree Models

Tree Models

- Let us see the tableux proof we did before again, for the formula

  $\varphi = \neg\langle R\rangle p \wedge (\langle R\rangle q \wedge \langle R\rangle p)$

$s{:}(\neg\langle R\rangle p \wedge (\langle R\rangle q \wedge \langle R\rangle p))$

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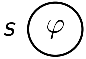

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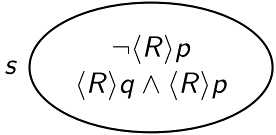

$$\begin{gathered} s{:}(\neg\langle R\rangle p \wedge (\langle R\rangle q \wedge \langle R\rangle p)) \\ s{:}\neg\langle R\rangle p \\ s{:}(\langle R\rangle q \wedge \langle R\rangle p) \end{gathered}$$

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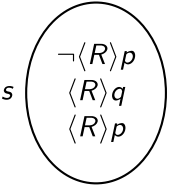

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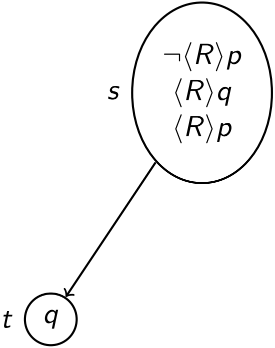

$$
\begin{array}{c}
s{:}(\neg\langle R\rangle p \land (\langle R\rangle q \land \langle R\rangle p)) \\
s{:}\neg\langle R\rangle p \\
s{:}(\langle R\rangle q \land \langle R\rangle p) \\
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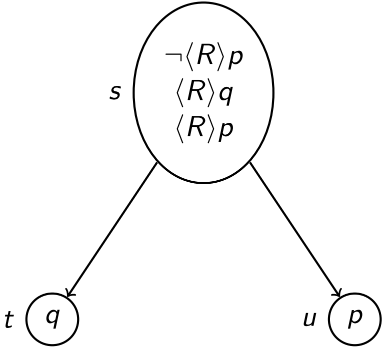

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s{:}(\neg\langle R\rangle p \wedge (\langle R\rangle q \wedge \langle R\rangle p)) \\
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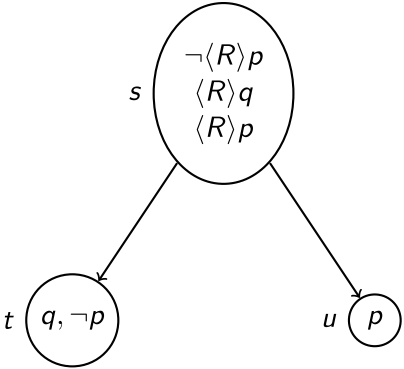

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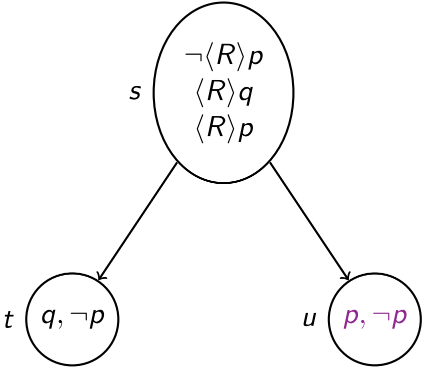

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Tree and Finite Model Properties

Tree and Finite Model Properties

- Using the rules of the tableaux calculus we only explore finite, tree models.

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- Then the $\langle R \rangle$-language
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Theorem: A formula in the $\langle R \rangle$-language is satisfialle if and only if it is satisfiable in a finite, tree relational structure.

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What we Covered in this Lecture

- We introduce a tableaux method to check satisfiability for the language with $\langle R \rangle$.
- We saw that we can use labels to describe what is going on in each point of a relational structure.
- More importantly: we saw that tableaux are a way to sistematically explore relational structures.
- Actually, from the tableaux algorithm we could learn some model properties: we only need to consider finite tree models.

Relevant Bibliography I

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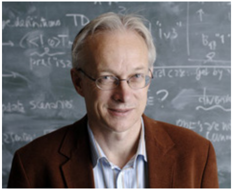

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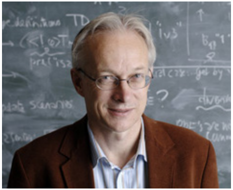

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- Some based on other algorithms:
  - MSpass (translation based) `http://www.cs.man.ac.uk/~schmidt/mspass/`
  - HyLoRes (resolution based) `http://trac.loria.fr/projects/hylores`

The Next Lecture

No Way to Say Warm in French