Logics for Computation

Lecture #6: No Way to Say Warm in French

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Nancy, France

ESSLLI 2008 - Hamburg - Germany

The Story so Far

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- We saw that the lenguage cannot say
  - "I am not a tree"
  - "I am infinite"
- On the positive side
  - It has a simple reasoning calculus: labelled tableaux
  - It is decidable.

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- We will learn to say "it won't take forever".

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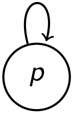

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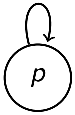

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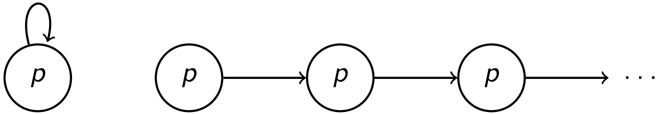

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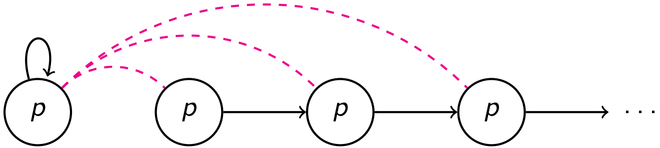

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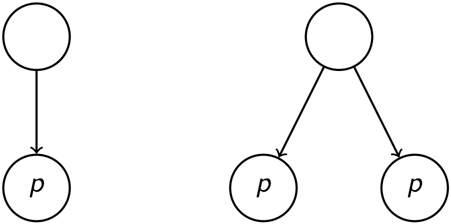
- We saw that the $\langle R \rangle$ language cannot distinguish between one and two!!!



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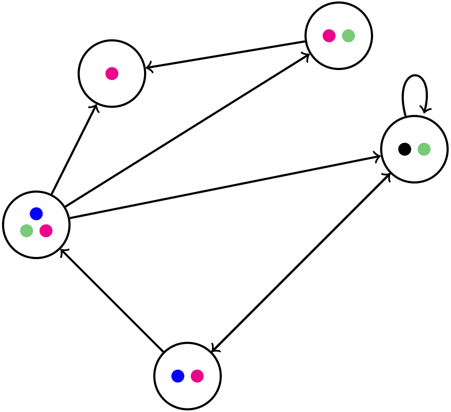

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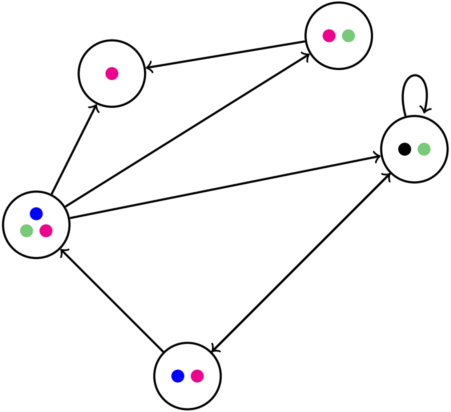
$\langle R \rangle$green $\wedge$ $\langle R \rangle$green



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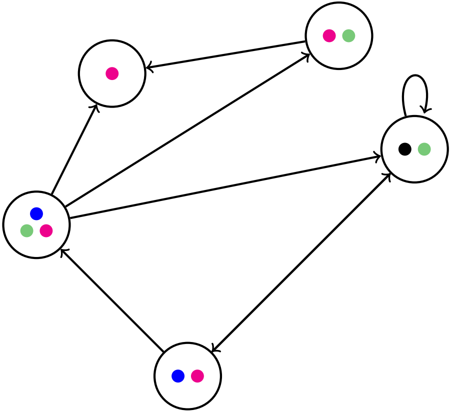
$\langle R \rangle(\text{green} \wedge \text{pink}) \wedge$
$\langle R \rangle(\text{green} \wedge \neg\, \text{pink})$



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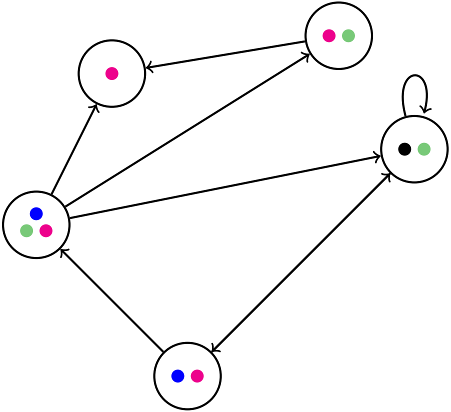
$\langle = 2\ R \rangle$ green



Alice in Wonderland

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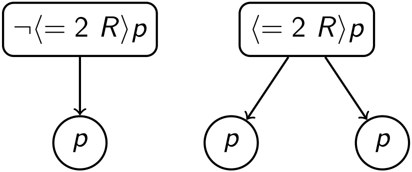

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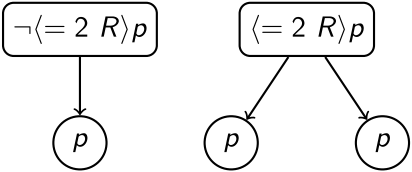

The models are not the same for the $\langle = n\ R\rangle$ language.

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- Let's get to work...

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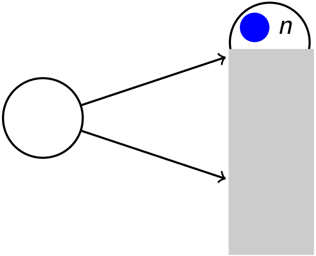

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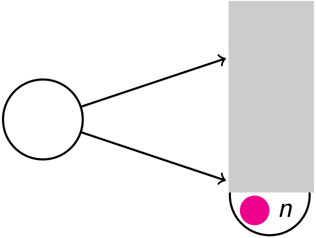

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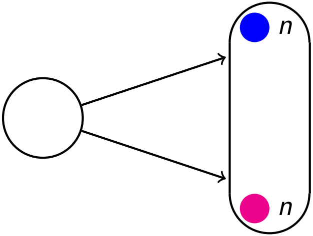

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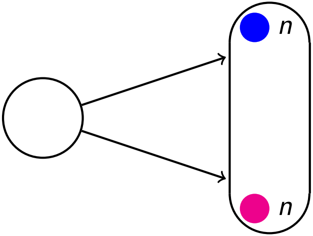

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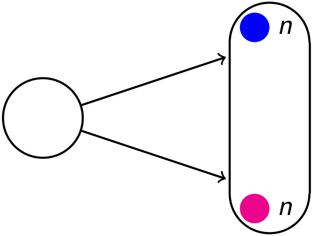

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- We can introduce names into our language (you probably know them as constants).

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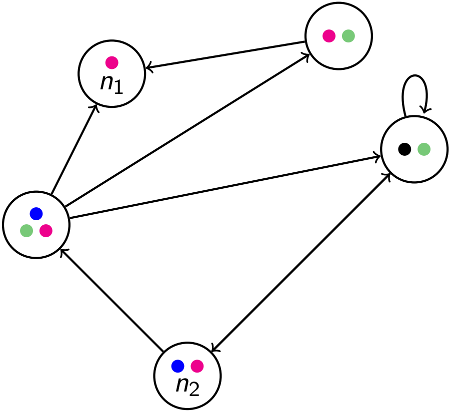

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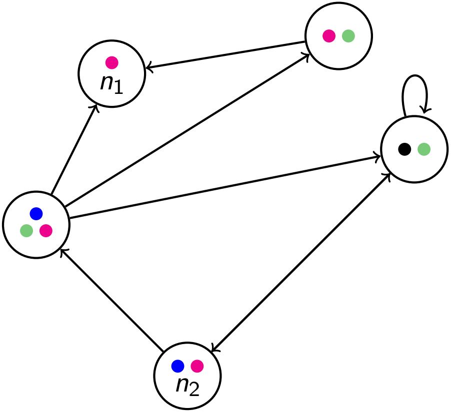

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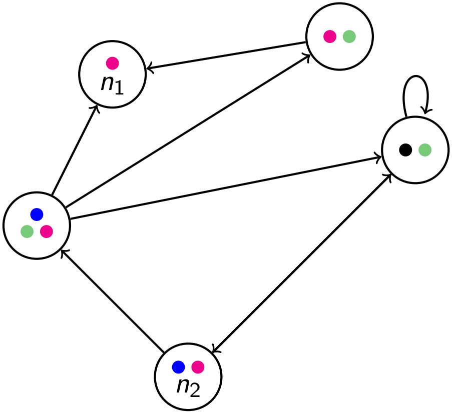

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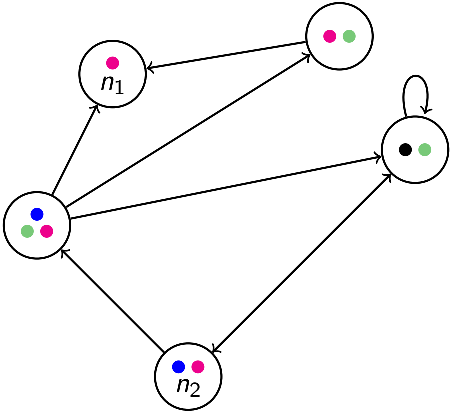

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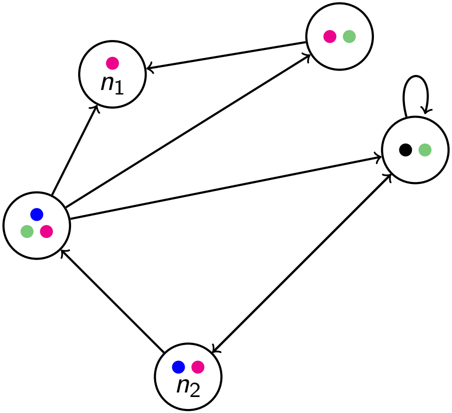

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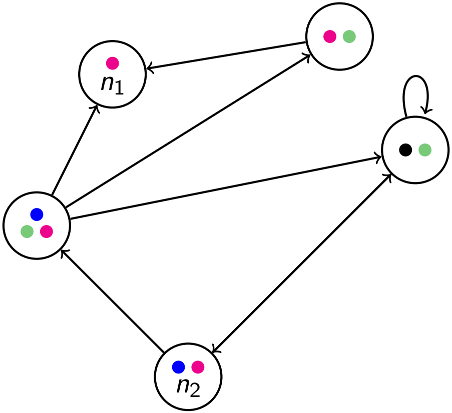

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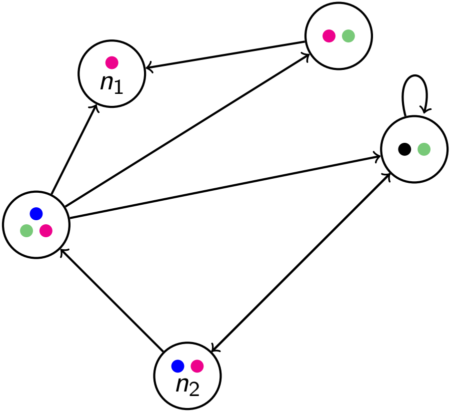

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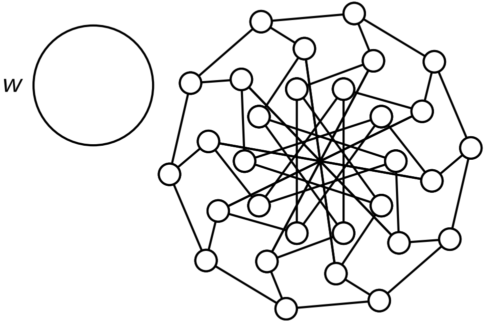

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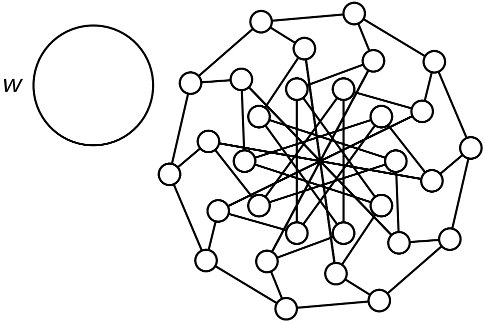

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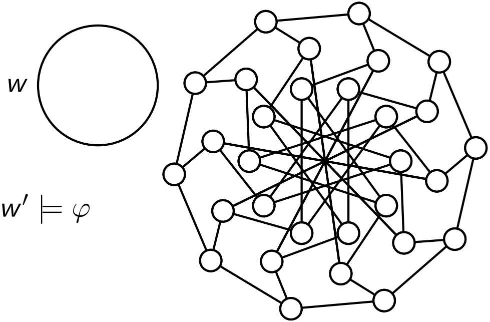

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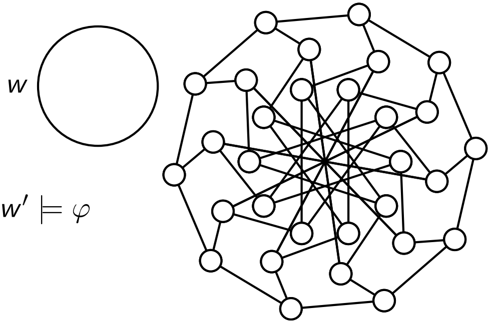

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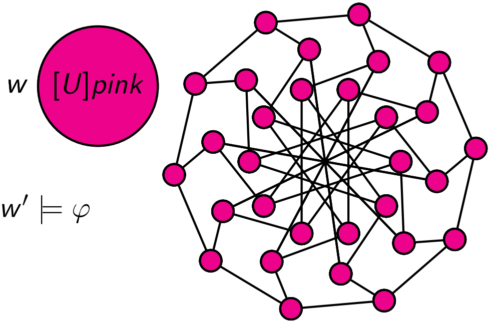

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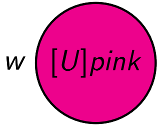

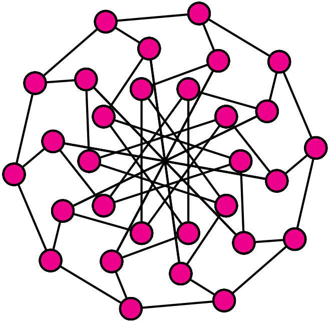

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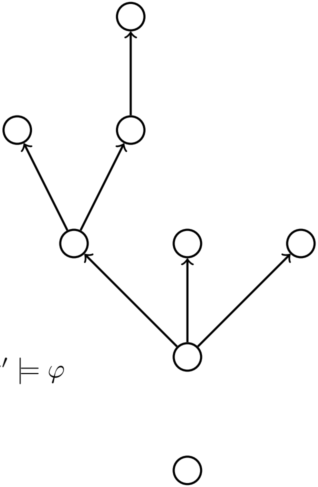

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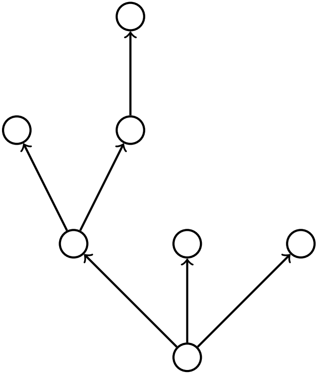

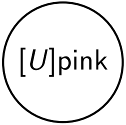

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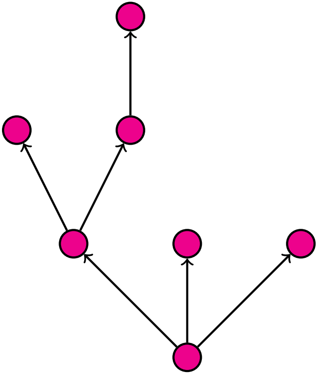

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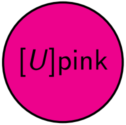

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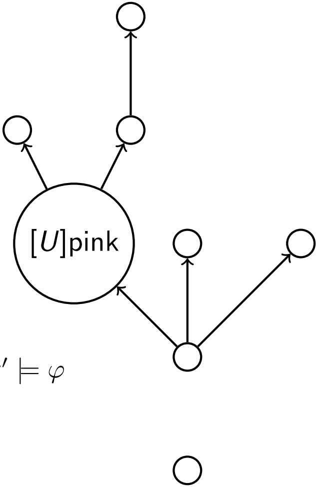

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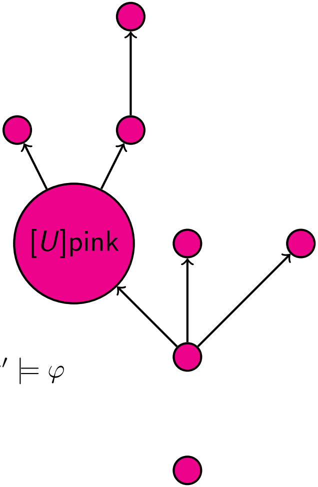

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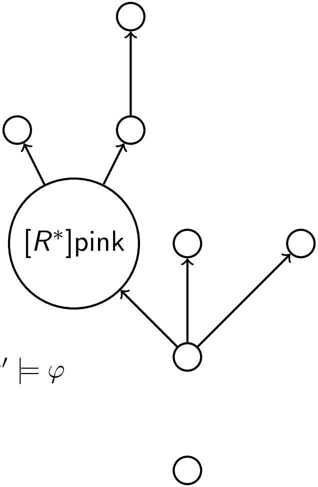

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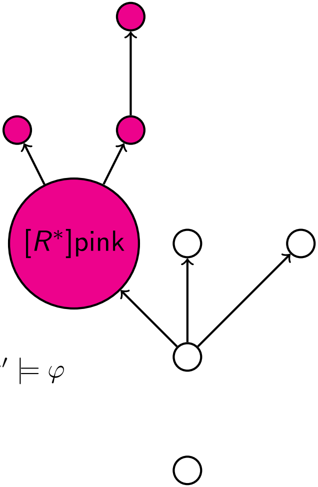

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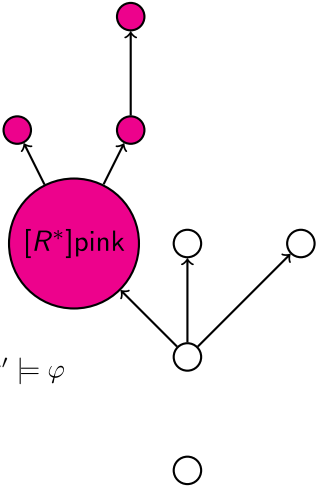

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- They are just different ways of talking about something.

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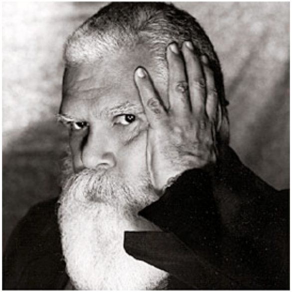

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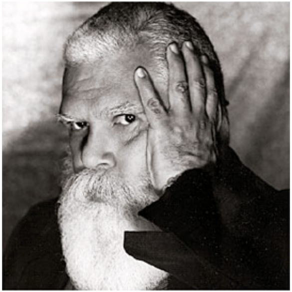

- My French is not good enough to say if it's true. . .
- But it's definitely a great science fiction book!

Delany, Samuel (1966). *Babel-17*. Ace Books.

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Blackburn, Patrick and van Benthem, J (2006). *Chapter 1 of the Handbook of Modal Logics*, Blackburn, P.; Wolter, F.; and van Benthem, J., editors, Elsevier.

de Rijke, Maarten (1993). *Extending Modal Logic* PhD Thesis. Institute for Logic, Language and Computation, Unviersity of Amsterdam.

The Next Lecture

DIY First Order Logic