# Additional Exercises: Logics and Statistics for Language Modeling 2009-2010

## 1 First-Order Logic

- Construct a single first-order model that satisfies the following formulas:

$$\forall xyz.((R(x,y) \land R(y,z)) \rightarrow R(z,z))$$
$$\exists xy.(P(x) \land \neg P(y))$$
$$\forall x.\exists y.(R(x,y) \land x \neq y)$$

- Construct a single first-order model that satisfies the following formulas:

$$\forall x.\exists y.R(x,y)$$
$$\exists xy.x \neq y$$
$$\forall xy.(R(x,y) \rightarrow \neg R(y,x))$$
$$\forall x.(P(x) \lor Q(x))$$
$$\forall x.(P(x) \rightarrow \neg \exists y.R(x,y))$$

- Construct a single first-order model that satisfies the following formulas:

$$\exists xyz.(\mathrm{Man}(x) \land \mathrm{Woman}(y) \land \mathrm{MarriedWith}(x,y) \land \mathrm{DogOf}(x,z))$$
$$\forall xy.(\mathrm{MarriedWith}(x,y) \rightarrow \mathrm{MarriedWith}(y,x))$$
$$\forall xy.(\mathrm{DogOf}(x,y) \rightarrow \mathrm{Dog}(y))$$
$$\forall x.(\mathrm{Dog}(x) \rightarrow \mathrm{has}(x, \mathrm{tail}(x)))$$
$$\forall x.(\mathrm{Dog}(x) \rightarrow \exists y.\mathrm{Loyal}(x,y))$$

- Construct a single first-order model that satisfies the following formulas:

$$\exists xyz.(\mathrm{Man}(x) \land \mathrm{Tall}(x) \land \mathrm{MarriedWith}(x,y) \land \mathrm{DogOf}(y,z))$$
$$\forall xy.(\mathrm{MarriedWith}(x,y) \rightarrow \mathrm{MarriedWith}(y,x))$$
$$\forall xyz.((\mathrm{DogOf}(x,z) \land \mathrm{MarriedWith}(x,y)) \rightarrow \mathrm{DogOf}(y,z))$$
$$\forall xy.(\mathrm{DogOf}(x,y) \rightarrow \mathrm{Dog}(y))$$
$$\forall x.(\mathrm{Dog}(x) \rightarrow \mathrm{has}(x, \mathrm{tail}(x)))$$

- Try to unify the following terms, where we use capital letters for variables (all the others are functions or contstants), and show the unifier substitution when successful.

1. $a(b,c) =^? a(X,Y)$
2. $a(X, c(d,X)) =^? a(2, c(d,Y))$
3. $a(X,Y) =^? a(b(c,Y), Z)$
4. $\mathit{tree}(\mathit{left}, \mathit{root}, \mathit{Right}) =^? \mathit{tree}(\mathit{left}, \mathit{root}, \mathit{tree}(a, b, \mathit{tree}(c, d, e)))$
5. $p(X, g(X), Y) =^? p(a, g(X), b)$
6. $p(1,2) =^? q(X,Y)$
7. $p(a,X) =^? p(X, g(X))$
8. $p(X) =^? p(g(X))$
9. $p(h(X)) =^? p(g(X))$
10. $q(X,Y) =^? q(Y,X)$
11. $q(1,Y) =^? q(Y,X)$
12. $q(g(X,Y), f(X)) =^? q(g(h(Z),Y), f(h(Z)))$
13. $q(g(X,Y), f(X)) =^? q(g(h(a),Y), f(h(Z)))$
14. $p(f(X, g(a)), g(g(Y))) =^? p(f(X,Z), g(Z))$
15. $p(f(X), Z) =^? p(Y, z)$
16. $r(f(X,X)) =^? r(f(g(Y),Y))$
17. $\mathit{app}(\mathit{cons}(X,Y), \mathit{nil}) =^? \mathit{app}(Z,Y)$
18. $p(X,Z) =^? p(f(Z), a)$

- Show that $\varphi$ does not entails $\psi$ by producing, in each case, a model where the left formula is true and the right formula is false.

| $\varphi$ | | $\psi$ |
|---|---|---|
| $\exists x.P(x,x)$ | $\not\models$ | $P(a,b)$ |
| $\exists x.P(x,x)$ | $\not\models$ | $P(a,a)$ |
| $\forall x.P(f(x), g(x))$ | $\not\models$ | $P(f(a), a)$ |
| $\forall x.P(f(x), f(g(x)))$ | $\not\models$ | $P(f(a), f(g(b)))$ |

- Given the following set of formulas:

$$\forall xy.(R(x,y) \vee R(y,x))$$
$$\exists xy.(P(x) \wedge \neg P(y))$$
$$\forall xy.(P(x) \rightarrow \neg R(x,y))$$
$$\forall xy.(R(x,y) \rightarrow R(y,x))$$

  – Apply Skolemization and obtain the corresponding set of clauses.
  – Use resolution to prove that the set is inconsistent.

- Given the following set of formulas:

$$\forall xy.(R(x,y) \rightarrow (P(y) \vee Q(y)))$$
$$\forall xy.(R(x,y) \rightarrow \neg P(y))$$
$$\exists xy.(R(x,y) \wedge \neg Q(y))$$

  – Apply Skolemization and obtain the corresponding set of clauses.
  – Use resolution to prove that the set is inconsistent.