

# Additional Exercises: Logics and Statistics for Language Modeling 2009-2010

## 1 Propositional Logics

- Classify the following formulas in
  - satisfiable: they are true in at least one valuation,
  - unsatisfiable: they are true in no valuation,
  - tautologies: they are true in all valuations,
  - contingent: they are true in some valuations and false in others
- 1.  $((p \wedge (p \rightarrow r)) \rightarrow (q \rightarrow r))$
- 2.  $p \vee q$
- 3.  $((p \vee q) \rightarrow r) \leftrightarrow (\neg p \wedge \neg q)$
- 4.  $(p \wedge (q \vee r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$
- We say that a formula  $\varphi$  *entails*  $\psi$ , if whenever  $\varphi$  is true then  $\psi$  is also true (equivalently, whenever the formula  $(\varphi \rightarrow \psi)$  is true in all situations).  
For the following pairs of formulas, when does  $A$  entails  $B$ ?, when does  $B$  entails  $A$ ?, when are both true?, when is none?
- 1.  $A = (p \wedge q), B = (p \vee q)$
- 2.  $A = (p \rightarrow q), B = (q \rightarrow p)$
- 3.  $A = ((p \vee q) \rightarrow r), B = (p \rightarrow r)$
- 4.  $A = ((p \rightarrow q) \rightarrow r), B = ((p \vee q) \rightarrow r)$
- 5.  $A = (p \rightarrow q), B = (p \leftrightarrow q)$
- Show using DP that the following formulas are tautologies:
  - 1.  $(\neg a \rightarrow b) \rightarrow ((a \rightarrow b) \rightarrow b)$
  - 2.  $(a \rightarrow (b \rightarrow c)) \leftrightarrow ((a \wedge b) \rightarrow c)$
  - 3.  $(b \rightarrow c) \rightarrow ((a \rightarrow b) \rightarrow (a \rightarrow c))$