Logics and Statistics for Language Modeling

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Today's Program

- Description Logics
- History and Applications
- Syntax and Semantics
- The Tableaux Method

Description Logics

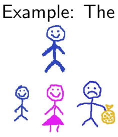
- Description Logics (DL) are formal languages which are specially tailored for knowledge representation.
- They originate from the Quillian's Semantic Networks and Minsky's Frame paradigm.
- Their main characteristics are:
  - A simple to use language (an extension of the propositional language, without variables);
  - But that includes a notion of quantification (guarded quantification);
  - With special operators chosen to facilitate the enunciation of definitions;
  - With a good balance between expressivity and tractability;
  - With highly optimized inference systems.

Description Logics

In a DL we have operators to build definitions using individuals, concepts and roles:

- Individuals are "objects" in a given universe.
- Concepts correspond to "classes of objects" and will be interpreted as sets in a given universe.
- Roles correspond to "links between objects" and will be interpreted as binary relations over a given universe.

Example: The "Happy Father"



Concepts = { Man, Woman, Happy, Rich }
Roles = { has-children }
Individuals = { carlos }
HappyFather ≡ Man ∧ ∃ has-children.Man ∧
∃ has-children.Woman ∧
∀ has-children.(Happy ∨ Rich)
carlos:¬HappyFather

Application Areas

- Terminological Knowledge Bases and Ontologies
  - DLs were created exactly for this task
  - Specially useful as a language to define and maintain ontologies
- Semantic Web
  - To add 'semantic markup' to the information in the web.
  - Such markup would use ontological repositories as a store of common definitions with clear semantics
  - DL inference systems would be used for the development, mantainment and merging of these ontologies, and for the dynamic evolution of resources (e.g. search).
- Computational Linguistics
  - Many tasks in computational linguistics require inference and 'background knowledge': reference resolution, question/answering.
  - In some cases, the expressive power of DLs is enough and we don't need to move to FOL.

Short Story of DL

- 1st Stage:
  - Incomplete Systems (Back, Classic, Loom, . . .)
  - Based in structural algorithms
- 2nd Stage:
  - Development of tableaux algorithms and first complexity results
  - Tableaux based systems for PSpace complete logics (Kris, Crack)
  - Research in optimization techniques
- 3rd Stage:
  - Tableaux algorithms for very expressive DLs
  - Tableau based systems with many optimizations for ExpTime Logics (FaCT, DLP, Racer, Pellet)
  - Relation with modal logics and fragments of FOL
- 4th Stage:
  - Mature implementations (Commercial!)
  - Applications and tools start to be widely used (e.g., Semantic Web).

Description Logics

HappyFather ≡ Man∧
∃ has-children.Man ∧
∃ has-children.Woman ∧
∀ has-children.(Happy ∨ Rich)
carlos:¬HappyFather

- The language is defined in three steps.
  - Concepts: we construct complex concepts using other concepts (atomics or introduced via definitions) and roles: E.g., ∃has-children.Man
  - Definitions: we use concepts to build definitions (or relations between definitions): E.g., HappyFather ≡ . . .
  - Assertions: assign concepts and roles to particular elements in our model: E.g., carlos:¬HappyFather

Concept Construction

- A concept can be
  - ⊤, the trivial concept, of which every element is a member.
  - An atomic concept: Man, Woman
  - Boolean Operators: If C and D are concepts the the following are concepts

$C \wedge D$	the conjunction of C and D	Rich ∧ Handsome
$C \vee D$	the disjunction of C and D	Rich ∨ Handsome
$\neg C$	the negation of C	¬Rich

  - Relational Operators: if C is a concept and R is a role, the following are concepts

$\forall R.C$	each element acc. through R is in C	∀has-children.Woman
$\exists R.C$	some element acc. through R is in C	∃has-children.Woman

Building Definitions

Given two concepts C and D, there are two types of definitions:

- Partial Definitions: $C \sqsubseteq D$. conditions specified in C are sufficient to qualify elements in C as members of D D, but they are not necessary; or vice-versa.

$$\exists\text{has-children.Man} \wedge \exists\text{has-children.Woman} \sqsubseteq \text{BusyFather} \quad \text{(suff. condition)}$$
$$\text{BusyFather} \sqsubseteq \exists\text{has-children.}\top \quad \text{(nec. condition)}$$

- Total Definitions: $C \equiv D$. Conditions indicated in D are both necessary and sufficient to qualify elements of D as elements of C (and vice-versa). Concepts C and D are equivalent.

$$\text{GrandMother} \equiv \text{Woman} \wedge \exists\text{has-children.}\exists\text{has-children.}\top$$

(Equivalent to say both $C \sqsubseteq D$ and $D \sqsubseteq C$)

Building Assertions

We can "assign assertions" to particular elements in the situation we are describing.
Given elements a and b, a concept C and a relation R

- Assigning elements to concepts: $a{:}C$. Indicates that C is true of a. I.e., all conditions indicated in C apply to a.

$$\text{carlos:Argentine}$$
$$\text{carlos:}(\text{Argentine} \wedge \exists\text{Lives-in.Europe})$$

- Assigning elements elements to relations: $(a, b){:}R$. Indicates that the elements a and b are related via the role R.

$$(\text{carlos,nancy})\text{:Lives-in}$$

A Complete Example

$$\begin{array}{rcl} \text{Woman} & \sqsubseteq & \text{Person} \wedge \exists\text{sex.Female} \\ \text{Man} & \sqsubseteq & \text{Person} \wedge \exists\text{sex.Male} \\ \text{FatherOrMother} & \equiv & \text{Person} \wedge \exists\text{has-children.Person} \\ \text{Mother} & \equiv & \text{Woman} \wedge \text{FatherOrMother} \\ \text{Father} & \equiv & \text{Man} \wedge \text{FatherOrMother} \end{array}$$

$$\text{alice:Mother}$$
$$(\text{alice,betty})\text{:has-children}$$
$$(\text{alice,carlos})\text{:has-children}$$

Reasoning with Description Logics

- There are different Reasoning Task that we might be interested in, when using Description Logics.
- For example:
  - Concept Inconsistency: Given a concept C, is C always empty in every model?
    Equivalently, can we find a model where C is not empty?
  - Concept Membership: Given some definitions T, some assertions A, a concept C and an individual a, does the information in $\langle T, A\rangle$ makes a a C?
    Equivalently, does every model where $\langle T, A\rangle$ is true, also makes $a : C$ true?
  - Concept Equivalence: Given some definitions T, some assertions A, and two concepts C_1 and C_2, does the information in $\langle T, A\rangle$ makes the concept $C_1 and C_2$ equivalent?
    Equivalently, does every model where $\langle T, A\rangle$ is true, also makes $C_1 \equiv C_2$ true?

The Tableaux Method

- We will use the Tableaux Method to solve the inference tasks we introduced in the previous slide.
- What is a tableaux? It's a method to search for models
  - It's a collection of formulas (assertions) organized as a tree.
  - Each branch of the tree represent a (partial) model of the root formula.
  - Branches are expanded via tableaux rules.
  - If a branch contains a contradiction it is closed.
  - If no further rule can be applied and there is at least a branch which is not closed, then we have found a model for the root.
- A branch is closed if for some C and some a, both $a : C$ and $a : \neg C$ are in the branch; or if $a : \neg\top$ is in the branch.

Tableaux Rules

For Conjunction:

$$\frac{a : C_1 \wedge C_2}{\begin{array}{c} a : C_2 \\ a : C_1 \end{array}} (\wedge) \qquad \frac{a : \neg(C_1 \wedge C_2)}{a : \neg C_1 \quad | \quad a : \neg C_2} (\neg\wedge)$$

For Disjunction

$$\frac{a : C_1 \vee C_2}{a : C_1 \quad | \quad a : C_2} (\vee) \qquad \frac{a : \neg(C_1 \vee C_2)}{\begin{array}{c} a : \neg C_2 \\ a : \neg C_1 \end{array}} (\neg\vee)$$

Tableaux Rules

For Existential:

$$\frac{a : \exists R.C}{\begin{array}{c} b : C \\ (a, b) : R \end{array}} (\exists) \qquad \frac{\begin{array}{c} a : \neg(\exists R.C) \\ (a, b) : R \end{array}}{b : \neg C} (\neg\exists)$$

for b a new individual

For Universal:

$$\frac{\begin{array}{c} a : \forall R.C \\ (a, b) : R \end{array}}{b : C} (\forall) \qquad \frac{a : \neg(\forall R.C)}{\begin{array}{c} b : \neg C \\ (a, b) : R \end{array}} (\neg\forall)$$

for b a new individual

Tableaux Rules

For a set T of Definitions

$$\frac{C_1 \sqsubseteq C_2 \in T}{a : \neg C_1 \vee C_2} (\sqsubseteq) \qquad \frac{C_1 \equiv C_2 \in T}{\begin{array}{c} a : \neg C_2 \vee C_1 \\ a : \neg C_1 \vee C_2 \end{array}} (\equiv)$$

for a any individual in the tableaux
For a set A of Assertions

$$\frac{a : C \in A}{a : C} (a :) \qquad \frac{(a, b) : R \in A}{(a, b) : R} ((a, b) :)$$

Using the Tableaux Rules

- Concept Inconsistency: Given a concept C, is C always empty in every model?
  Run the tableaux rules on $a : C$ for an arbitrary a. If all the branches are closed, then C is always empty in every model.
- We prove that $C \wedge \neg(D \vee C)$ is inconsistent.

$$a : C \wedge \neg(D \vee C)$$
$$a : C$$
$$a : \neg(D \vee C)$$
$$a : \neg D$$
$$a : \neg C$$
$$\otimes$$

Using the Tableaux Rules

- Concept Membership: Given some definitions T, some assertions A, a concept C and an individual a, does the information in $\langle T, A\rangle$ makes a a C?
  Run the tableaux rules on $a : \neg C$. If all the branches are closed, then in every model $a : C$.
- We prove that given $T = \{\mathsf{Father} \equiv \mathsf{Man} \wedge \exists \mathsf{has\text{-}child}.\top\}$ and $A = \{\mathsf{a} : \mathsf{Father}\}$ it follows that $\mathsf{a} : \mathsf{Man}$.

$\mathsf{a} : \neg\mathsf{Man}$
$\mathsf{a} : \mathsf{Father}$
$\mathsf{a} : \neg\mathsf{Father} \vee (\mathsf{Man} \wedge \exists\mathsf{has\text{-}child}.\top)$
$\mathsf{a} : \mathsf{Father} \vee \neg(\mathsf{Man} \wedge \exists\mathsf{has\text{-}child}.\top)$

Left branch: $\mathsf{a} : \neg\mathsf{Father}$, $\otimes$

Right branch: $\mathsf{a} : (\mathsf{Man} \wedge \exists\mathsf{has\text{-}child}.\top)$, $\mathsf{a} : \mathsf{Man}$, $\mathsf{a} : \exists\mathsf{has\text{-}child}.\top$, $\otimes$

Using the Tableaux Rules

- Concept Equivalence: Given some definitions T, some assertions A, and two concepts C_1 and C_2, does the information in $\langle T, A\rangle$ makes the concept $C_1 and C_2$ equivalent?
  Run the tableaux rules on $a : C_1 \wedge \neg C_2$. If all the branches are closed, then in every model $C_1 \sqsubseteq C_2$.
  Do the same for $a : C_1 \wedge \neg C_2$.

Exercises

Prove that, with respect to the following definitions,

$$\begin{aligned}
\mathsf{Man} &\equiv \mathsf{Male} \wedge \mathsf{Human}\\
\mathsf{Parent} &\equiv \exists\mathsf{children}.\top\\
\mathsf{Father} &\equiv \mathsf{Man} \wedge \mathsf{Parent}\\
\mathsf{Father\text{-}with\text{-}only\text{-}male\text{-}children} &\equiv \mathsf{Father} \wedge \mathsf{Human} \wedge (\forall\mathsf{children}.\mathsf{Male})\\
\mathsf{Father\text{-}with\text{-}only\text{-}sons} &\equiv \mathsf{Man} \wedge (\exists\mathsf{children}.\top) \wedge (\forall\mathsf{children}.\mathsf{Man})
\end{aligned}$$

the concept Father-with-only-sons and Father-with-only-male-children are **not** equivalent.