Logics and Statistics for Language Modeling

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Today's Program

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- Description Logics
  - Syntax and Semantics
  - The Tableaux Method
  - Inference Tasks

Description Logics: Syntax

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- Roles: $\mathsf{ROL} = \{R_1, R_2, \ldots, R_n, \ldots\}$
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for C_1, C_2 concepts.
- Assertions:
$$i : C \mid (i_1, i_2) : R$$
for C a concept, i, i_1, i_2 individuals and R a role.

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- Let φ be a formula in DL. Then φ is satisfiable iff

$$M, g \models T_x(\varphi)$$

for M a first-order model for the signature $\langle \text{VARS}, \text{IND}, \{\}, \text{CON} \cup \text{ROL} \rangle$ and g an arbitrary assignment.

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- A branch is closed if for some C and some i, both $i : C$ and $i : \neg C$ are in the branch; or if $i : \neg\top$ is in the branch.

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For Negation: $\frac{a : \neg\neg C}{a : C} \ (\neg)$

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For a set A of Assertions

$$\frac{a : C \in A}{a : C}\ (a :) \qquad \frac{(a, b) : R \in A}{(a, b) : R}\ ((a, b) :)$$

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  Run the tableaux rules on $a : C$ for an arbitrary a. If all the branches are closed, then C is always empty in every model.
- We prove that $C \wedge \neg(D \vee C)$ is inconsistent.

$$a : C \wedge \neg(D \vee C)$$
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$$\otimes$$

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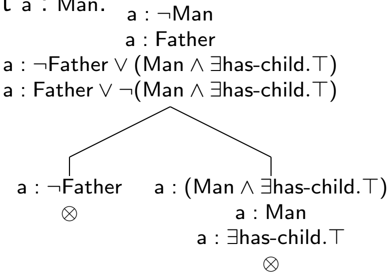
- We prove that given $T = \{\text{Father} \equiv \text{Man} \wedge \exists\text{has-child}.\top\}$ and $A = \{\text{a} : \text{Father}\}$ it follows that a : Man.



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  Run the tableaux rules on $a : C_1 \wedge \neg C_2$. If all the branches are closed, then in every model $C_1 \sqsubseteq C_2$.
  Do the same for $a : C_1 \wedge \neg C_2$.

Exercises

Prove that, with respect to the following definitions,

$$
\begin{aligned}
\text{Man} &\equiv \text{Male} \wedge \text{Human} \\
\text{Parent} &\equiv \exists \text{children}.\top \\
\text{Father} &\equiv \text{Man} \wedge \text{Parent} \\
\text{Father-with-only-male-children} &\equiv \text{Father} \wedge \text{Human} \wedge (\forall \text{children}.\text{Male}) \\
\text{Father-with-only-sons} &\equiv \text{Man} \wedge (\exists \text{children}.\top) \wedge (\forall \text{children}.\text{Man})
\end{aligned}
$$

the concept Father-with-only-sons and Father-with-only-male-children are **not** equivalent.