Worksheet

Completeness

Exercise 1. Let $\mathcal{M}^c = \langle W^c, R^c, V^c \rangle$ be the canonical model for $\mathbf{K}$, and let $w \in W^c$. Suppose that $\Diamond\varphi \in w$. Let $v^- = \{\varphi\} \cup \{\psi \mid \Box\psi \in w\}$.

1. Prove that v^- is consistent.
2. Let v be a maximal consisten extending v^-. Prove that $wR^c v$.

Exercise 2. Let $\mathbf{K4}$ be the axiomatic system $\mathbf{K}$ extended with the axiom $\Diamond\Diamond p \to \Diamond p$. Let $\mathcal{M}^c$ be the cannonical model for $\mathbf{K4}$. Prove that R^c is transitive.

Exercise 3. Prove that when a pure formula is valid in a frame, it defines a first order property over a signature without unary predicate symbols. Hint, use ST.

Exercise 4. Let $\mathcal{M} = \langle W, R, V \rangle$ be named and φ pure. Prove that if for all pure instances ψ of φ, $\mathcal{M} \models \psi$, then $\langle W, R \rangle \models \psi$.

Exercise 5. Prove that the formula Bridge: $\Diamond i \wedge @_i p \to \Diamond p$ is a theorem of K_h (i.e., give a syntactic proof in the axiomatic system). Hint, prove the contrapositive using the modal theorem $(\Diamond q \wedge \Box p) \to \Diamond(q \wedge p)$ and the Introduction and Back axioms).

Exercise 6. Let Γ be a K_h-MCS. for each nominal i we defined $\Delta_i = \{\varphi \mid @_i\varphi \in \Gamma\}$. Prove that

1. If $k \in \Gamma$ then $\Delta_k = \Gamma$.
2. Δ_i is a K_h-MCS that contains i.
3. For any nominal i, j, if $i \in \Delta_j$ then $\Delta_i = \Delta_j$.

Exercise 7. Let Δ be a normal hybrid logic extending $\mathbf{K}_h$+R. Let Σ be a Δ-MCS and $\mathcal{M}^c = \langle W^c, R^c, V^c \rangle$ be the canonical model we defined for Σ.

- Prove that $\mathcal{M}^c$ is a hybrid model, i.e., prove that nominals are interpreted as singletons.
- Prove the Truth Lemma for $\mathcal{M}^c$.