Worksheet

Description Logics

Exercise 1. Express the following sentences in terms of the description logic $\mathcal{ALC}$:

1. All employees are humans.
2. A mother is a female who has a child.
3. A parent is a mother or a father.
4. A grandmother is a mother who has a child who is a parent.
5. Only humans have children that are humans.

Exercise 2.

1. Construct a TBox describing a university (5 to 10 definitions). Use concept names such as University, Faculty, Education, Lecturer, Student, and role names such as teaches, works-for and studies-at.
2. Extend the TBox from the previous item to a knowledge base by constructing an appropriate ABox (5 assertions).

Exercise 3. Show using an $\mathcal{ALC}$ constraint system that $a{:}(\exists r.E)$ is a logical consequence of the knowledge base $K = \langle \{C \sqsubseteq \exists r.D, D \sqsubseteq E \sqcup F, F \sqsubseteq E\}, \{a{:}C\}\rangle$.

Exercise 4. Show, using an $\mathcal{ALCN}$ constraint system that the following concept

$$(\geq 3 \text{ friend}) \sqcap (\leq 3 \text{ friend}) \sqcap (\exists \text{friend.Nice}) \sqcap (\forall \text{friend.}((\text{Nice} \sqcap \neg\text{Cool}) \sqcup (\neg\text{Nice} \sqcap \text{Cool}))$$

is satisfiable.

Exercise 5. Find a model for the following knowledge base with TBox ($r_1 \circ r_2$ is role composition, $\neg r$ is role complementation):

$$\begin{array}{l}
\text{Donkey} \sqsubseteq \forall \text{eats.}(\text{Plant} \sqcup \{\text{sven}\}) \\
\text{Lion} \sqsubseteq \neg\text{Donkey} \sqcap \forall \text{eats.}(\text{Donkey} \sqcup \{\text{sven}\}) \\
\text{Plant} \sqsubseteq \neg\text{Donkey} \sqcap \neg\text{Lion} \\
\text{Human} \sqsubseteq \neg\text{Donkey} \sqcap \neg\text{Lion} \sqcap \neg\text{Plant} \\
\text{eats} \sqsubseteq \neg\text{friends} \\
\text{eats} \circ \text{eats} \sqsubseteq \text{eats} \\
\top \sqsubseteq \exists \text{eats.}\top
\end{array}$$

and ABox:

$$\begin{array}{l}
\text{sven:}(\text{Human} \sqcap \geq 1 \text{ eats}^-) \\
\text{harald:Plant} \\
\text{dane:Donkey} \\
\text{lorette:Lion}
\end{array}$$

Can lorette and sven be friends?

Exercise 6. Given the two concept definitions

$$\begin{array}{rcl} \text{BinaryTree} & \equiv & \leq 2 \text{ hasBranch} \sqcap \forall \text{hasBranch.BinaryTree} \\ \text{List} & \equiv & \leq 1 \text{ hasBranch} \sqcap \forall \text{hasBranch.List} \end{array}$$

Prove that for any model M, we have $M \models \text{List} \sqsubseteq \text{BinaryTree}$.

Exercise 7. Prove or disprove the following, for the description logic $\mathcal{ALC}$:

1. There is a TBox that has no models at all.
2. There is a TBox that has only finite models.
3. Every TBox has either no models at all or infinitely many models.
4. There is a TBox T such that all models of T are either infinite or contain a cycle (when viewed as a graph).

Exercise 8. Prove or disprove the following, for the description logic $\mathcal{ALC}$:

1. There is an ABox that has no models at all.
2. There is an ABox that has only finite models.
3. Every ABox has either no model or infinitely many models.
4. There is an ABox A such that all models of A contain a cycle (when viewed as a graph).