Hybrid Logics
Undecidability and infinite models

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What we cover in this lecture

- Today we will see some examples of modal logics which are undecidable.
- We will show some techniques used to prove undecidability, and other related properties.

The logic $\mathcal{HL}(\downarrow)$

- We already discussed it would be interesting to have dynamic naming, or "variables" in addition to nominals.
- Suposse we can create names "on the fly". Introduce the $\downarrow x$, that names the current state x.
- $\downarrow x$ names the current evaluation point, and let us refer to it in the rest of the formula. E.g., $\downarrow x.\Diamond x$ characterizes reflexive points.

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To the signature of the basic modal logic add an infinite enumerable set of variables $\textsc{var}$.

Syntax of $\mathcal{HL}(\downarrow)$

$$\varphi ::= x \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \langle r \rangle \varphi \mid \downarrow x.\varphi$$

where $p \in \textsc{prop}$, $r \in \textsc{rel}$, $x \in \textsc{var}$.

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The semantics of this logics is defined over usual Kripke models $\mathcal{M} = \langle W, \{R_i\}, V \rangle$ plus a assignment function $g : \textsc{var} \to W$ which assigns variables to elements in the domain.

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- Given a model $\mathcal{M} = \langle W, \{R_i\}, V\rangle$ and an assignament g define:

$\mathcal{HL}(\downarrow)$ Semantics:

$$
\begin{array}{rcl}
\mathcal{M}, g, w \models x & \text{iff} & g(x) = w \\
\mathcal{M}, g, w \models \downarrow x.\varphi & \text{iff} & \mathcal{M}, g^x_w, w \models \varphi \text{ where } g^x_w \text{ is identical to } g \\
 & & \text{except } g^x_w(x) = w.
\end{array}
$$

Infinite model

- We saw that the basic modal logic (and other extensions) have the finite model property
- This is useful for proving decidability (knowing also a bound for the model size)
- We will see that $\mathcal{HL}(\downarrow)$ is able to force an infinite model
- By itself, this does not prove undecidability, but it paves the way

Infinite model

- Write a formula which says that there is a non empty set B whose elements constitute a strict partial order. That is:
  - Irreflexive
  - Transitive
- And where every element has a successor

This implies that B is infinite.

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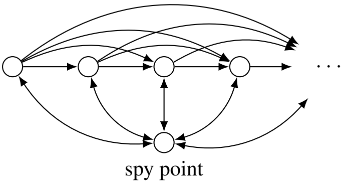

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Let φ be the formula $\mathit{Back} \wedge \mathit{Spy} \wedge \mathit{Irr} \wedge \mathit{Succ} \wedge \mathit{Tran}$

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Proof. By construction of φ.

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We have to check also that φ does have models.

Theorem

There exists $\mathcal{M}, g, w$ s.t. $\mathcal{M}, g, w \models \varphi$.

Proof. Let B be an infinite set of elements and w an element such that $w \notin B$. Let R be the smallest relation such that

- R defines an strict partial order over B
- wRb and bRw for every element $b \in B$

$\mathcal{M} = \langle B \cup \{w\}, R, V\rangle$ verifies $\mathcal{M}, g, w \models \varphi$ (for any V and g).

Undecidability

- How do we prove that a logic is undecidable?
- If we want to prove it in a direct way, we must write a formula that codifies arbitrary executions in a Turing machine
- The tiling problem, which has been proved to be undecidable, will be useful for the modal case

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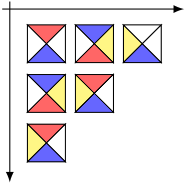

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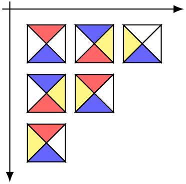

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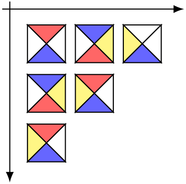
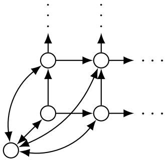


- The tiling problem in $\mathbb{N} \times \mathbb{N}$ is known to be undecidable
- Given a set of types of tiles $\mathcal{T}$, we want to write a formula $\varphi_{\mathcal{T}}$ such that $\varphi_{\mathcal{T}}$ is satisfiable iff there exists a tiling for $\mathcal{T}$

The tiling problem for $\mathcal{HL}(\downarrow)$

- We will use again a spy point



- (Notice, codifying the tiling problem does not imply forcing an infinite model, why?)

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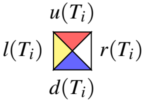

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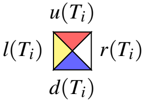
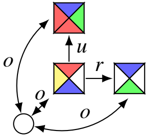


- Suppose that we also have a modality $\langle o \rangle$ that we use to move from the spy point into every tile
- And we have as well modalities $\langle u \rangle$ and $\langle r \rangle$ in order to move up from a tile and to the right of a tile respectively.



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There is concluence between up-right and right-up

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Finally, there is a tile in each point of the grid and all the colors coincide:

Each point has a single type of tile

$$(\mathit{Unique}) \quad [o]\left(\bigvee_{1\leq i\leq n} t_i \wedge \bigwedge_{1\leq i<j\leq n}(t_i \rightarrow \neg t_j)\right)$$

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Each tile has an adjacent tile above which is appropriately colored

$$(\textit{Vert}) \quad [o]\bigwedge_{1\leq i\leq n}\left(t_i \to \langle u\rangle \bigvee_{1\leq j\leq n, u(T_i)=d(T_j)} t_j\right)$$

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Let

$$\varphi_T = \downarrow s.(\mathit{Back} \wedge \mathit{Empty} \wedge \mathit{Spy} \wedge \mathit{Grid} \wedge \mathit{Func} \wedge \mathit{Conf} \wedge \mathit{Unique} \wedge \mathit{Vert} \wedge \mathit{Horiz})$$

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($\Leftarrow$) Suppose that $f : \mathbb{N} \times \mathbb{N} \to T$ is a tiling in $\mathbb{N} \times \mathbb{N}$. We define the model $\mathcal{M} = \langle W, \{R_o, R_u, R_r\}, V\rangle$:

- $W = \mathbb{N} \times \mathbb{N} \cup \{w\}$
- $R_o = \{(w, v), (v, w) \mid v \in \mathbb{N} \times \mathbb{N}\}$
- $R_u = \{(x, y), (x, y + 1) \mid x, y \in \mathbb{N}\}$
- $R_r = \{(x, y), (x + 1, y) \mid x, y \in \mathbb{N}\}$
- $V(t_i) = \{x \mid x \in \mathbb{N} \times \mathbb{N}, f(x) = T_i\}$

It is not hard to see that $\mathcal{M}, w \models \varphi_T$

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  - The “two person corridor tiling” is EXPTIME-complete

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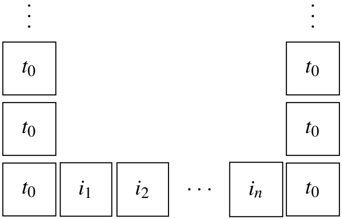
- The game starts with the referee putting tiles as follows



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  - If after a finite number of rounds a tile of type t_{s+1} is put in the first column, Spoiler wins
  - Otherwise (if no player can make a valid move, t_{s+1} is not in the column 1, or the game goes on infinitely) Duplicator wins

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- When do the players win or lose?
  - If after a finite number of rounds a tile of type t_{s+1} is put in the first column, Spoiler wins
  - Otherwise (if no player can make a valid move, t_{s+1} is not in the column 1, or the game goes on infinitely) Duplicator wins
- The problem of detecting if Spoiler has a winning strategy is known to be EXPTIME-complete.

Two person corridor tiling

- Using the “two person corridor tiling” one can prove that PDL is EXPTIME-hard, codifying the tree of possible moves between Spoiler y Duplicator.
- Also, it can be used to prove that $K + A$ is EXPTIME-hard.