LogicS

Lecture #5: About Trees, and How to Cut Them

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- We have discussed when two models are the same.
- We have seen an algorithm to check whether a formula is true in a given model.

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- What about the $\langle R \rangle$ language?

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- The solution is: labels!
- They will help us keep track of what is going on in each point in our model.

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s{:}(\langle R\rangle p \wedge (\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q))) \\
s{:}\langle R\rangle p \\
s{:}\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q) \\
s{:}\neg\langle R\rangle\neg q \\
s{:}\neg\langle R\rangle(p \wedge q) \\
sRt \\
t{:}p \\
t{:}\neg\neg q \\
t{:}q \\
t{:}\neg(p \wedge q) \\
t{:}\neg p \qquad\qquad t{:}\neg q
\end{array}$$

The Complete Cast, plus an Example

$$\frac{s{:}(\varphi \land \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

$$\frac{s{:}\neg(\varphi \land \psi)}{s{:}\neg\varphi \qquad\qquad s{:}\neg\psi}$$

$$\frac{s{:}\neg\neg\varphi}{s{:}\varphi}$$

$$\frac{s{:}\langle R\rangle\varphi}{\begin{array}{c} sRt \\ t{:}\varphi \end{array}}$$

for t a new label

$$\frac{\begin{array}{c} s{:}\neg\langle R\rangle\varphi \\ sRt \end{array}}{t{:}\neg\varphi}$$

$$\begin{array}{c}
s{:}(\langle R\rangle p \land (\neg\langle R\rangle\neg q \land \neg\langle R\rangle(p \land q))) \\
s{:}\langle R\rangle p \\
s{:}\neg\langle R\rangle\neg q \land \neg\langle R\rangle(p \land q) \\
s{:}\neg\langle R\rangle\neg q \\
s{:}\neg\langle R\rangle(p \land q) \\
sRt \\
t{:}p \\
t{:}\neg\neg q \\
t{:}q \\
t{:}\neg(p \land q) \\
t{:}\neg p \qquad\qquad t{:}\neg q \\
\text{closed} \qquad\qquad \text{closed}
\end{array}$$

The Complete Cast, plus an Example

$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

$$\frac{s{:}\neg(\varphi \wedge \psi)}{s{:}\neg\varphi \qquad\qquad s{:}\neg\psi}$$

$$\frac{s{:}\neg\neg\varphi}{s{:}\varphi}$$

$$\frac{s{:}\langle R\rangle\varphi}{\begin{array}{c} sRt \\ t{:}\varphi \end{array}}$$

for t a new label

$$\frac{\begin{array}{c} s{:}\neg\langle R\rangle\varphi \\ sRt \end{array}}{t{:}\neg\varphi}$$

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s{:}(\langle R\rangle p \wedge (\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q))) \\
s{:}\langle R\rangle p \\
s{:}\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q) \\
s{:}\neg\langle R\rangle\neg q \\
s{:}\neg\langle R\rangle(p \wedge q) \\
sRt \\
t{:}p \\
t{:}\neg\neg q \\
t{:}q \\
t{:}\neg(p \wedge q) \\
\begin{array}{cc} t{:}\neg p & t{:}\neg q \\ \text{closed} & \text{closed} \end{array}
\end{array}$$

- Which are the similarities/differences with tableaux for PL?

The Complete Cast, plus an Example

$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

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$$\frac{s{:}\langle R\rangle\varphi}{\begin{array}{c} sRt \\ t{:}\varphi \end{array}}$$

for t a new label

$$\frac{\begin{array}{c} s{:}\neg\langle R\rangle\varphi \\ sRt \end{array}}{t{:}\neg\varphi}$$

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s{:}\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q) \\
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- Which are the similarities/differences with tableaux for PL?
- How do we know that we got it right?

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- Which are the similarities/differences with tableaux for PL?
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A Closer Look

A Closer Look

- Which similarities / differences with tableaux for PL?

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$$\frac{s{:}\langle R\rangle\varphi}{\begin{array}{c} sRt \\ t{:}\varphi \end{array}}$$

for t a new label

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A Closer Look

- Which similarities / differences with tableaux for PL?
  - Does the calculus terminate?

$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}}$$

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A Closer Look

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A Closer Look

- Which similarities / differences with tableaux for PL?
  - Does the calculus terminate?
  - What are labels? What are they doing? Can we use them?
  - Is this an algorithm?

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- Did we get it right?

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- Did we get it right?
  - Did we get it right in the PL case, to start with?! Consider the rule:

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- What can we learn from the calculus?

A Closer Look

- Which similarities / differences with tableaux for PL?
  - Does the calculus terminate?
  - What are labels? What are they doing? Can we use them?
  - Is this an algorithm?
  - Is it a good algorithm?
- Did we get it right?
  - Did we get it right in the PL case, to start with?! Consider the rule:
- What can we learn from the calculus?
  - Something about models!

$$\frac{s{:}(\varphi \wedge \psi)}{\begin{array}{c} s{:}\varphi \\ s{:}\psi \end{array}} \qquad \frac{s{:}\langle R\rangle\varphi}{\begin{array}{c} sRt \\ t{:}\varphi \end{array}}$$

for t a new label

$$\frac{s{:}\neg(\varphi \wedge \psi)}{s{:}\neg\varphi \qquad s{:}\neg\psi} \qquad \frac{\begin{array}{c} s{:}\neg\langle R\rangle\varphi \\ sRt \end{array}}{t{:}\neg\varphi}$$

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$$\frac{s{:}\neg(\varphi \wedge \psi)}{s{:}\neg\varphi \qquad \begin{array}{c} s{:}\varphi \\ s{:}\neg\psi \end{array}}$$

Tree Models

Tree Models

- Let us see the tableux proof we did before again, for the formula

$$\varphi = s{:}(\langle R\rangle p \wedge (\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q)))$$

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$$s{:}\langle R\rangle p$$
$$s{:}\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q)$$

s

$$\langle R\rangle p$$
$$\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q)$$

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- Let us see the tableux proof we did before again, for the formula

s

$$\langle R \rangle p$$
$$\neg\langle R \rangle \neg q$$
$$\neg\langle R \rangle (p \wedge q)$$

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$$s{:}\neg\langle R \rangle \neg q$$
$$s{:}\neg\langle R \rangle (p \wedge q)$$

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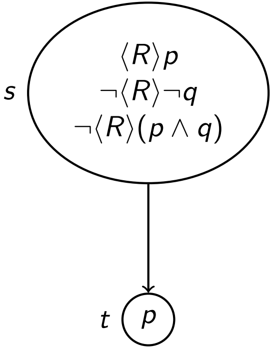

$$
\begin{gathered}
\varphi = s{:}(\langle R\rangle p \wedge (\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q))) \\
s{:}\langle R\rangle p \\
s{:}\neg\langle R\rangle\neg q \wedge \neg\langle R\rangle(p \wedge q) \\
s{:}\neg\langle R\rangle\neg q \\
s{:}\neg\langle R\rangle(p \wedge q) \\
sRt \\
t{:}p
\end{gathered}
$$

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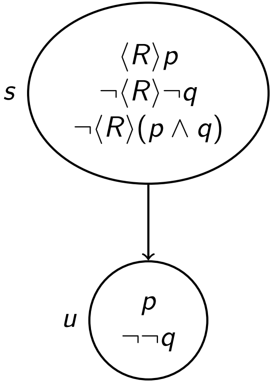

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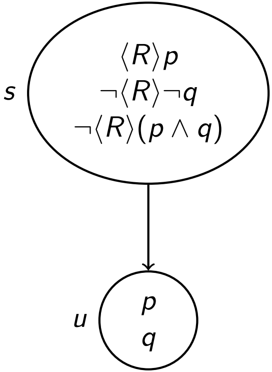

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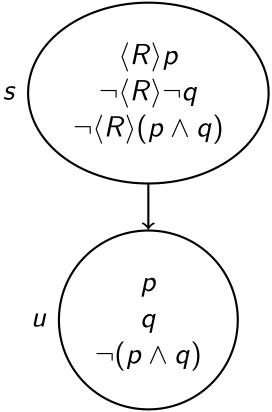

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$$sRt$$
$$t{:}p$$
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$$t{:}q$$
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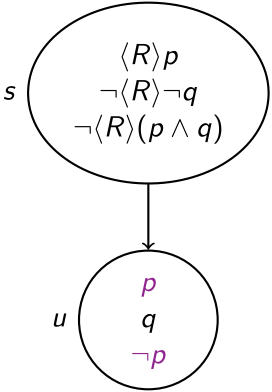

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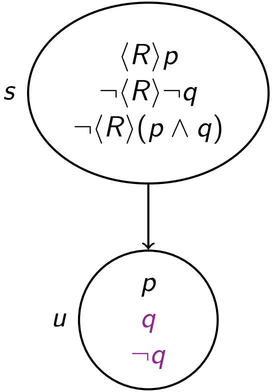

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Tree and Finite Model Properties

Tree and Finite Model Properties

- Using the rules of the tableaux calculus we only explore finite, tree models.

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for t a new label

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- Using the rules of the tableaux calculus we only explore finite, tree models.
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- Then the $\langle R \rangle$-language
  - cannot say infinite,
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$$\frac{s{:}\neg\neg\varphi}{s{:}\varphi}$$

Theorem: A formula in the $\langle R \rangle$-language is satisfialle if and only if it is satisfiable in a finite, tree relational structure.

What we Covered in this Lecture

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- We introduce a tableaux method to check satisfiability for the language with $\langle R \rangle$.

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What we Covered in this Lecture

- We introduce a tableaux method to check satisfiability for the language with $\langle R \rangle$.
- We saw that we can use labels to describe what is going on in each point of a relational structure.
- More importantly: we saw that tableaux are a way to systematically explore relational structures.
- Actually, from the tableaux algorithm we could learn some model properties: we only need to consider finite tree models.

Relevant Bibliography I

Tableaux Algorithms

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Fitting, Melvin (1983). *Proof Methods for Modal and Intuitionistic Logics*. D. Reidel Publishing Co., Dordrecht.

Relevant Bibliography II

Tree and Finite Model Properties

Relevant Bibliography II

Tree and Finite Model Properties

- Unraveling, the procedure to turn arbitrary models into trees, was introduced by Segerberg.

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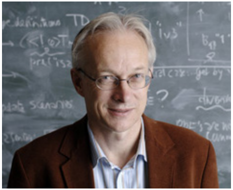

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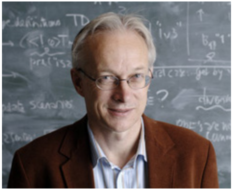

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Segerberg, Krister (1971). *An Essay in Classical Modal Logic*, Department of Philosophy Uppsala University, Sweden. Uppsala Philosophical Studies.

van Benthem, Johan (1985). *Modal Logic and Classical Logic*, Bibliopolis.

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- Some based on other algorithms:
  - MSpass (translation based) `http://www.cs.man.ac.uk/~schmidt/mspass/`
  - HyLoRes (resolution based) `http://www.glyc.dc.uba.ar/intohylo/hylores`

The Next Lecture

No Way to Say Warm in French