LogicS: A Modal Perspective

Spillover #1

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Inference tasks

The semantic perspective give us a good way to think about inference tasks. We shall concentrate on the following three:

- Given a certain logical description φ, and a model $\mathcal{M}$, does the formula correctly describe (some aspect of) the model? More simply: is the formulas true (or satisfied) in the model? This task is called the model checking task.
- Given a certain logical description φ, does there exist a model where that formula is true? This task is called the satisfiability checking task (or the model building task).
- Given a logical description φ, is it true (or satisfied) in all models? This task is called the validity checking task.

Satisfiability and Validity are Dual

- A formula φ is valid iff $\neg\varphi$ is not satisfiable.
- A consequence of this observation is: if we have a method for solving the satisfiability problem (that is, if we have an algorithm for building models) then we have a way of solving the validity problem.
- Why? Because: to test whether φ is valid, simply give $\neg\varphi$ to the algorithm for solving satisfiability. If it can't satisfy it, then φ is valid.
- Well, we have an algorithm for satisfiability (namely the tableaux method), so let's put this observation to work.

Expressivity

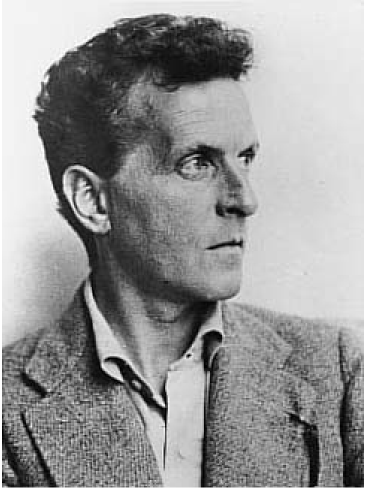
- As we remarked earlier, it is possible to think about the semantics of PL in terms of relational structure.
- This gives us a way of comparing the expressivity of PL with the more powerful logics we shall study later.
- The idea is simple: think of PL as a way of talking about one element (!) relational structures of the form $\langle \{d\}, \{P_1, \ldots, P_n\} \rangle$.
- That is, we have one individual, and one property for every propositional letter p_n (think of each P_i as a colour — we are covering the individual with coloured dots).
- This way of thinking about PL semantics is equivalent to the truth conditional semantics. Can you see why?
- That is, PL validity is completely determined by one element relational structures! Measured this way, its expressivity is low.

Computability

- We haven't directly said much about computability, but it should be clear that PL is a "computable logic".
- For a start, model checking is clearly computationally straightforward — it's linear in the length of the input formula.
- And checking satisfiability (and hence validity) is clearly computable too. The truth table method shows that we can do it in 2^n steps, where n is the number of propositional symbols in the input formal.
- Can we do better that 2^n steps? In particular, can the tableaux method do satisfiability/validity checking more efficiently.
- Sadly, it seems the answer is no.

Relevant Bibliography

- The truth table method was pioneered by the philosopher Ludwig Wittgenstein (1889–1951) in his first famous philosophical work, the “Tractatus Logico-Philosophicus”.
- He used the method in support of his celebrated “picture” theory of meaning.



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Emile Post →

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Charles Peirce →

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- The tableaux method in the form presented here is due to Raymond Smullyan, logician, magician, and puzzle-supremo.
- His classic exposition of the method is in his book "First-Order Logic" (1968) , which remains one of the best technical expositions of the subject.
- Was professor at Indiana University!