Introduction to Logic

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What is Logic? a Personal Perspective

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- What is Logic?

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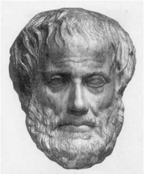
- What is Logic?
  - A tool to model how we think ???



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  What is Logic used for?

- I believe that a fair and acurate answer is:

  Logics are used to describe.

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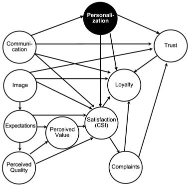

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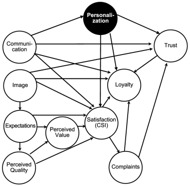

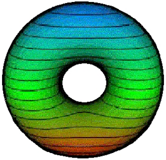

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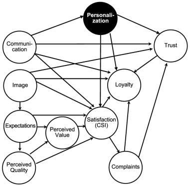

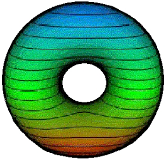

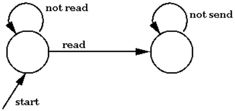

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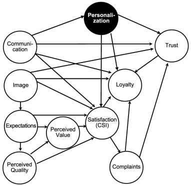

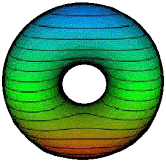

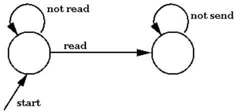

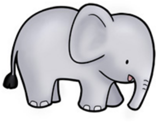

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- When/for what are logics better than drawing?

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- Which are the questions that we can pose a Logic?
- The most common one is probably satisfiability:

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- Let's write $M \models \varphi$ (and we say "M satisfies φ") if the model M makes the formula φ true.

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Can it be a validity? No.

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In most of the rest of this tutorial we will talk about this relation between models and formulas for one particular logic:

Propositional Logic

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- Propositions are the smallest (atomic) formulas of propositional logics. They can be combined using logical operators into complex formulas.

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- Logical Operators or Connectives:

$\neg$	$\neg p$	negation

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  - Precedence: if we declare that $\neg$ has higher precedence than $\wedge$ then $\neg p \wedge q$ represents $(\neg p) \wedge q$

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- We said that our atomic, not further analyzed, formulas were the propositional symbols. Hence, our models should explicitly decide whether propositional formulas are true or false.
- Def.: A propositional model is a mapping of propositional symbols to elements of the set $\{\text{True}, \text{False}\}$ (an assignment or valuation).

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r	True

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Propositional Logic: Semantics

- Examples of Models:

	M_1	M_2
p	True	True
q	False	True
r	True	True

We sometimes write these models as

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M(p) = \text{True or } \ldots
\end{array}
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- Writing things with NAND only can be cumbersome. But it is very much used, for example, to build logical circuits.
- Other examples of functionally complete sets are $\{\neg, \vee\}$ and $\{\neg, \wedge\}$.

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- Now that we know that $\{\neg, \vee\}$, for example, are functionally complete, we can give a final definition of $\models$.

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  Assume we are given a model M, we define

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- Which we can trivially turn into an algorithm Eval to check whether a formula φ is true in a model M.

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- In other words, if two models M_1 and M_2 assign the same truth values to the variables in φ they will both agree on the truth value of φ (independently of the other values).
- Third try: when looking for a model for φ we only need to inspect a finite number of models (at most 2^n for n the number of propositional variables in φ).

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- The algorithm Sat is called a decision procedure for satisfiability, because it always terminate with the correct answer to the question "Is φ satisfiable?"

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  Suppose we are evaluating the formula $(\neg p \wedge \neg q) \wedge \neg r$

p	q	r	$(\neg p \wedge \neg q) \wedge \neg q$
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
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- For example, our algorithm could realize already after the first step, that if we want a model for the whole formula then r should be set to False, and don't try setting it to True any more.

Semantic Tableaux

- Intuitively, tableaux are ways to systematically organize the search for a model of a formula.
- Tableaux are built by applying rules to an input formula. These rules break down the formula to detect all possible ways of building a model.
- Each branch of a tableaux records one way of trying to build a model. Some branches ("closed branches") don't lead to models. Others branching ("open branches") do.
- The best way to learn is via an example...

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$$\neg(p \wedge q) \wedge \neg\neg r$$
$$p$$

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$$\neg(p \wedge q)$$

$$\neg\neg r$$

$$r$$

$$\neg p \qquad\qquad\qquad\qquad \neg q$$

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$$\begin{array}{ccc}
 & (\neg(p \wedge q) \wedge \neg\neg r) \wedge p & \\
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 & p & \\
 & \neg(p \wedge q) & \\
 & \neg\neg r & \\
 & r & \\
\neg p & & \neg q
\end{array}$$

Contradiction!!!

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$$\frac{\neg(\varphi \wedge \psi)}{\neg\varphi \qquad\qquad \neg\psi}\ (\neg\wedge)$$

$$\frac{\neg\neg\varphi}{\varphi}\ (\neg\neg)$$

$$(\neg(p \wedge q) \wedge \neg\neg r) \wedge p$$

$$\neg(p \wedge q) \wedge \neg\neg r$$

$$p$$

$$\neg(p \wedge q)$$

$$\neg\neg r$$

$$r$$

$\neg p$	$\neg q$
Contradiction!!!	*Model*

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- And that they were better than pictures because we could query this descriptions and obtain answers.
- Let's see a concrete example of this. Suppose that you are charged with the sitting arrangement for the dinner tonight.
  - There are three chairs arranged in a line on one side of a table.
  - These chairs should be occupied by your Aunt, your Father and your Sister.
  - Sister doesn't want to sit in the middle chair
  - Aunty doesn't want to sit next to your Father.

Describing and Answering Questions

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- We will also discuss the limitations of propositional logics, and in which cases its expressiveness is too limited and present ways in which we can extend it.